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Pomeron calculus in zero transverse dimensions: Summation of pomeron loops and generating functional for multiparticle production processes

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Abstract. In this paper we address two problems in pomeron calculus in zero transverse dimensions: the summation of the pomeron loops and the calculation of the processes of multiparticle generation. We introduce a new generating functional for these processes and obtain the evolution equation for it. We argue that in the kinematic range given by $1 \ll \ln\left(1/\alpha_S^2\right) \ll \alpha_S Y \ll \frac{1}{\alpha_S}$, we can reduce the pomeron calculus to the exchange of non-interacting pomerons with the renormalized amplitude of their interaction with the target. Therefore, the summation of the pomeron loops can be performed using the Mueller–Patel–Salam–Iancu approximation.

1 Introduction

The problem of the pomeron interaction in zero transverse dimensions has been discussed in the framework of reggeon calculus [1–6] about three decades ago. However, recently, we have seen a revival of the interest in this problem (see [7–14] and references therein). The very reason for this in our opinion is related to the hope to solve the old problem of finding the high energy asymptotic behavior of the scattering amplitude in QCD . We hope for a solution, not in the mean field approximation, where the solution has been discussed and well understood both analytically and numerically [15–51], but in the approach where the so called pomeron loops should be taken into account [52–57]. The problem of taking into account the pomeron loops can be reduced to BFKL pomeron calculus [58–71] and/or to the solution of the statistical physics problems of the Langevin equation and directed percolation [72–77]. The latter approach is based on the probabilistic interpretation of the pomeron calculus, which also has roots in the past [78, 79].

Pomeron calculus in zero transverse dimensions, being an oversimplified model, has the same description in terms of directed percolation as the general approach. Thus solving this model we can gain experience that will be useful for the solution of the general problem of the interaction of the BFKL pomerons in QCD.

It is well known that the calculus in zero transverse dimensions can be treated as a system that evolves in imaginary time $it = Y$ with the Hamiltonian

$$H = -\Delta\phi\phi^+ + \lambda(\phi\phi^{+2} - \phi^2\phi^+)$$

and the evolution equation for the wave function

$$\frac{d\Psi}{dY} = -H\Psi\,, \tag{1}$$

where the pomeron intercept is $\Delta \propto \alpha_S$ and the triple pomeron vertex $\lambda \propto \alpha_S^2$.

In the next section we will discuss the evolution equation for the generating functional that describes the system of pomerons in terms of probabilities to find 'wee' partons (color dipoles [80, 81]). We introduce $\Gamma(1 \to 2) = \Delta$ and $\Gamma(2 \to 1) = \Delta\gamma$ with γ the amplitude for the low energy interaction of the color dipole with the target (target–pomeron vertex) and $\gamma \propto \alpha_S^2$. The estimates for Δ and λ are given in the leading order of perturbative QCD. We can trust the approach with the Hamiltonian of (1) only in the kinematic region of Y given by the following equation:

$$1 \ll \ln(1/\alpha_S^2) \ll \alpha_S Y \ll \frac{1}{\alpha_S}\,. \tag{2}$$

Indeed, the n pomeron exchanges give contributions proportional to $(\gamma e^{\Delta Y})^2$ and, therefore, we need to sum them in the kinematic region, where $\gamma e^{\Delta Y} \geq 1$ or $\Delta Y \geq \ln(1/\gamma) \approx \ln(1/\alpha_S^2)$. However, we cannot go to ultrahigh energies, since we do not know the higher order corrections to the BFKL kernel and to the triple pomeron vertex. The contribution of the BFKL pomeron exchange can be written as $\gamma e^{(\Delta + \text{Const}\,\alpha_S^2)Y}$ and the high order correction term is essential for $\alpha_S Y > 1/\alpha_S$.

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In this paper, for the kinematic range given by (2), we obtain two results. We introduce a new generating functional that allows us to calculate processes of multiparticle generation, since it gives us the probability to have a given number of cut and uncut pomerons [83]. We derive the evolution equation for this generating functional both in the mean field approach and in the approach that takes into account the pomeron loops. A similar problem was also considered by Ciafaloni and Marchesini [82] more than 30 years ago in the framework of the RFT Lagrangian. In the present paper we reproduce the general result of [82], in particular the independent dynamics in the one direction of rapidity of an uncut pomeron and a new object (a linear combination of an uncut and cut pomeron). This corresponds to partial diagonalization of the Lagrangian in terms of the new variable $\phi_+ + \phi_- - \mathrm{i}\phi_c$ found in [82].

The second result is related to the method of summation of the pomeron loops. We claim that in the kinematic region of (2) the pomeron interaction given by Hamiltonian of (1) can be reduced to the system of free pomerons with the renormalized amplitude of the dipole–target interaction at low energies. In other words, we can view the evolution of our system of 'wee' partons (color dipoles) as a system of non-interacting partons only with emission absorbed in the evolution of the BFKL pomerons, and all specific features of this system determined by the low energy amplitude of the 'wee' parton interaction with the target. Having this in mind we state that the Mueller–Patel–Salam–Iancu approach [85–88] gives the solution to the problem.

The paper is organized as follows. In the next section we introduce the mean field approach and discuss it in the framework of the generating functional. We introduce a new generating functional that gives us the possibility to calculate the probability to find a given number of cut and uncut pomerons. Therefore, knowing this generating functional we can calculate the cross section with given multiplicities. We derive the evolution equation for this functional. In Sect. 3 we take into account the pomeron loops and generalize the evolution equation. In this section we reconsider the problem of summation of all pomeron loops in the kinematic region of (2) and argue that we can reduce this problem to a consideration of a system of non-interacting pomerons with renormalized vertex of the pomeron–target interaction. Based on this idea we use the Mueller–Patel–Salam–Iancu approach to calculate the scattering amplitude at high energies both for elastic and inelastic interactions with different multiplicities of particles in the final state.

In conclusion, we summarize the results and discuss open problems.

2 The mean field approximation

2.1 General approach

Our approach to multiparticle production is based on the AGK cutting rules [83]. These rules stem from the unitarity constraint in the s-channel, namely,

$$2\,\mathrm{Im}\,A^{\mathrm{BFKL}}(s,b) = 2N^{\mathrm{BFKL}}(s,b) = G^{\mathrm{BFKL}}_{\mathrm{in}}(s,b)\,, \quad (3)$$

where $\mathrm{Im}\,A^{\mathrm{BFKL}}(s,b) \equiv N^{\mathrm{BFKL}}(s,b)$ denotes the imaginary part of the elastic scattering amplitude for the dipole–dipole interaction at energy $W = \sqrt{s}$ and at the impact parameter b. It is normalized in such a way that the total cross section is equal to $\sigma_{\mathrm{tot}} = 2\int \mathrm{d}^2 b N^{\mathrm{BFKL}}(s,b)$. $G^{\mathrm{BFKL}}_{\mathrm{in}}$ is the contribution of all inelastic processes for the pomeron and $\sigma_{\mathrm{in}} = \int \mathrm{d}^2 b G_{\mathrm{in}}(s,b)$. Therefore, (3) gives us the structure of the BFKL pomeron exchange through the inelastic processes, and it can be formulated as the statement that the exchange of the BFKL pomeron is related to the processes of multigluon production in a certain kinematics (see Fig. 1). Equation (3) is proven in [58, 59] in QCD. Using it, we can express all processes of multiparticle production in terms of exchange and interactions of the BFKL pomerons and/or the cut BFKL pomerons (see Fig. 2).

The AGK cutting rules establish the relation between different processes that stem from BFKL pomeron diagrams. For example, the simple triple pomeron diagrams in Fig. 3 lead to three inelastic processes: diffractive production of the system with mass $\ln(M^2/m^2) = Y - Y_0$ (Fig. 3A); multigluon production in the entire kinematic region of rapidity $Y - 0$ with the same multiplicity of gluons as in one pomeron (Fig. 3B); and multigluon production in the region $Y - 0$ but with the same multiplicity of gluons as in one pomeron only in the rapidity window $Y - Y_0$, while for the rapidity $Y_0 - 0$ the gluon multiplicity is two times larger than for one pomeron (Fig. 3C). The AGK cutting rules [83] say that the cross sections of these three processes are related as

$$\sigma_{\mathrm{A}} : \sigma_{\mathrm{B}} : \sigma_C = 1 : -4 : 2\,. \quad (4)$$

At first sight the cross section of process B is negative, but it should be stressed that one pomeron also contributes to the same process and the resulting contribution is always positive.

Figures 1 and 3 as well as (4) allow us to understand the equation for single diffractive production in the mean

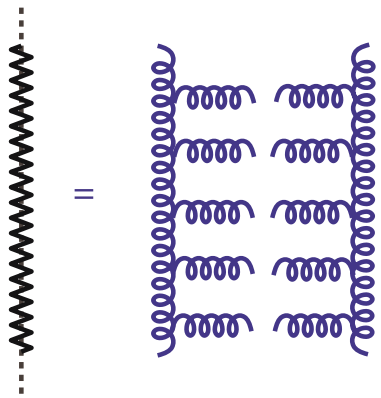

Fig. 1. The diagram for the cut BFKL pomeron that illustrates (3)

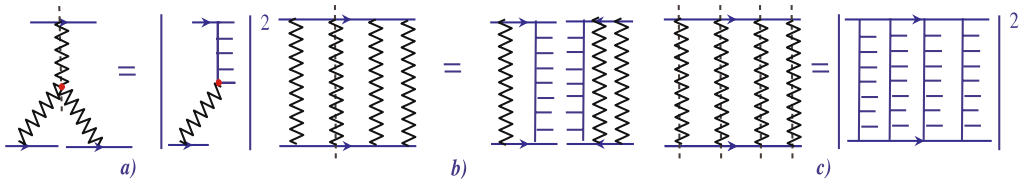

Fig. 2. Several examples of pomeron diagrams that contribute to multiparticle production: diffraction production of the bunch of particles in the region of rapidity $\ln(M^2/m^2) = Y - Y_0$ (**a**); the process of multiparticle production with the average multiplicity due to exchange of four pomerons (**b**); and the process of multiparticle production with the multiplicity four times larger than the average multiplicity

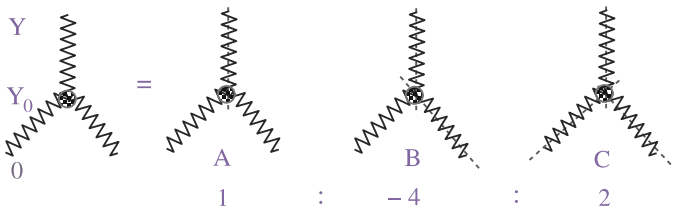

Fig. 3. AGK cutting rules for the triple pomeron diagram: diffractive production of the system with mass $\ln(M^2/m^2) = Y - Y_0$ (A); multigluon production in the entire kinematic region of rapidity $Y - 0$ with the same multiplicity of gluons as in one pomeron (B); and multigluon production in the region $Y - 0$ but with the same multiplicity of gluons as in one pomeron only in the rapidity window $Y - Y_0$, while for the rapidity $Y_0 - 0$ the gluon multiplicity is two times larger than for one pomeron (C)

field approximation that has been written by Kovchegov and Levin [89] and that has the following form (see Fig. 4 for a graphical representation of this equation):

$$\begin{aligned}
&\frac{\partial N^{\mathrm{D}}(\mathbf{x}_{01},\mathbf{b},Y,Y_0)}{\partial Y} \\
&= \frac{\alpha C_F}{\pi^2}\int_\rho \mathrm{d}^2x_2\,\frac{x_{01}^2}{x_{02}^2x_{12}^2} \\
&\quad\times\left(N^{\mathrm{D}}\left(\mathbf{x}_{02},\mathbf{b}+\frac{1}{2}\mathbf{x}_{12},Y,Y_0\right)\right. \\
&\quad+N^{\mathrm{D}}\left(\mathbf{x}_{12},\mathbf{b}+\frac{1}{2}\mathbf{x}_{02},Y,Y_0\right)-N^{\mathrm{D}}(\mathbf{x}_{01},\mathbf{b},Y,Y_0) \\
&\quad+N^{\mathrm{D}}\left(\mathbf{x}_{02},\mathbf{b}+\frac{1}{2}\mathbf{x}_{12},Y,Y_0\right)N^{\mathrm{D}}\left(\mathbf{x}_{12},\frac{1}{2}\mathbf{x}_{02},Y,Y_0\right) \\
&\quad-4N^{\mathrm{D}}\left(\mathbf{x}_{02},\mathbf{b}+\frac{1}{2}\mathbf{x}_{12},Y,Y_0\right)N_0\left(\mathbf{x}_{12},\mathbf{b}+\frac{1}{2}\mathbf{x}_{02},Y\right) \\
&\quad\left.+2N_0\left(\mathbf{x}_{02},\mathbf{b}+\frac{1}{2}\mathbf{x}_{12},Y\right)N_0\left(\mathbf{x}_{12},\mathbf{b}+\frac{1}{2}\mathbf{x}_{02},Y\right)\right),
\end{aligned} \tag{5}$$

with the initial condition given by

$$N^{\mathrm{D}}(\mathbf{x}_\perp,\mathbf{b},Y=Y_0,Y_0) = N_0^2(\mathbf{x}_\perp,\mathbf{b},Y_0)\,, \tag{6}$$

where N_0 is the solution of the Balitsky–Kovchegov equation [21–23] and $N^{\mathrm{D}}(\mathbf{x},\mathbf{b};Y;Y_0)$ is the diffraction dissociation of the colorless dipole with size x at impact parameter b into the system of gluon with the rapidity gap larger than Y_0 at energy Y. At first sight, (5) contradicts the AGK relations given by (4) (see Fig. 3), but this contradiction can easily be resolved if we take into account the coefficient 2 in (3) (see [89] for more details as well as for a proof based directly on the dipole approach to high energy scattering).

Despite the simple structure of (5), which is only a little bit more complicated than the Balitsky–Kovchegov equation, as far as we know there exists no analytical solution to this equation and there is only an attempt to solve it numerically [92, 93]. However, this equation has a simple solution in the toy model [8, 89, 94], which we are going to discuss.

2.2 Pomeron calculus in zero transverse dimensions: general approach

The mean field approximation looks extremely simple in pomeron calculus in zero transverse dimensions (the toy model [80, 81, 90, 91]). Indeed, in the toy model, in which there is no dependence on the sizes of the interacting dipoles, the generating functional degenerates to the gener-

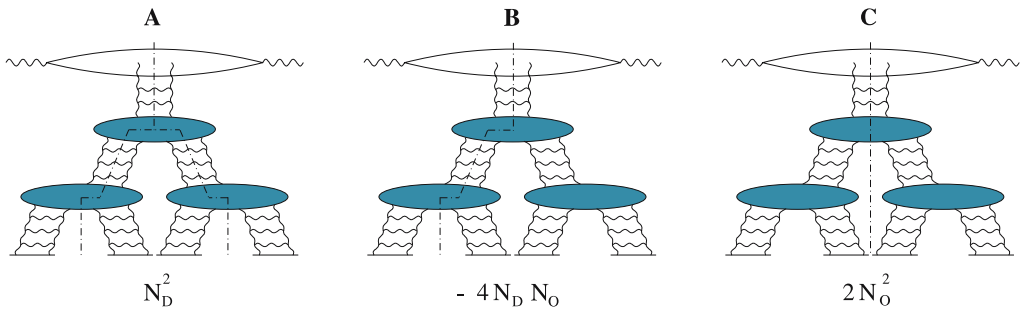

Fig. 4. Different pomeron cuts contributing to the cross section of diffractive dissociation, which lead to different terms on the right hand side in (5)

ating function of the form

$$Z_0(u|Y) = \sum_{n=0}^{\infty} P_n(Y) u^n \,, \tag{7}$$

where $P_n(Y)$ is the probability to find n dipoles (and/or n pomerons) at rapidity Y. For the probabilities $P_n(Y)$ we can write the Markov chain, namely [80, 81, 90, 91]

$$\frac{\mathrm{d}P_n(Y)}{\mathrm{d}Y} = -\Gamma(1\to 2) n P_n(Y) + \Gamma(1\to 2)(n-1)P_{n-1}(Y) \,. \tag{8}$$

Equation (8) has a simple structure: for the process of dipole splitting we see two terms. The first one, with the negative sign, describes a decrease of the probability P_n due to the process of the splitting of dipoles. The second term, with positive sign, is responsible for an increase of the probability due to the same processes of dipole interactions.

Equation (8) can be rewritten in the elegant form of the master equation for Z_0, namely

$$\frac{\partial Z_0(u|Y)}{\partial Y} = -\Gamma(1\to 2) u(1-u)\frac{\partial Z_0(u|Y)}{\partial u} \,, \tag{9}$$

where $\Gamma(1\to 2) \propto \bar{\alpha}_{\mathrm{S}}$ in QCD. The initial and boundary conditions look as follows:

$$\begin{aligned} &\text{initial condition: } Z_0(u|Y=0) = u \,, \\ &\text{boundary condition: } Z_0(u=1|Y) = 1 \,. \end{aligned} \tag{10}$$

In (10) the initial condition means that we are studying the evolution of one dipole, while the boundary condition follows from the normalization of the sum of probabilities. With this initial condition the linear differential equation (9) can be written as a non-linear one:

$$\frac{\partial Z_0(u|Y)}{\partial Y} = -\Gamma(1\to 2) Z_0(u|Y) + \Gamma(1\to 2) Z_0^2(u|Y) \,. \tag{11}$$

Introducing the scattering amplitude,

$$N_0(\gamma|Y) = \mathrm{Im}\, A_{\mathrm{el}} = -\sum_{n=1}^{\infty} \frac{(-1)^n}{n!} \frac{\partial^n Z_0(u|Y)}{\partial u^n}\bigg|_{u=1} \gamma^n \,, \tag{12}$$

with γ being the scattering amplitude of the interaction of a single dipole with the target, we can find (9) for the amplitude [95]:

$$\frac{\partial N_0(\gamma|Y)}{\partial Y} = \Gamma(1\to 2)(\gamma-\gamma^2)\frac{\partial N_0(\gamma|Y)}{\partial \gamma} \,. \tag{13}$$

Using the initial condition $N_0(\gamma|Y=0) = \gamma$ one can rewrite (13) as a non-linear equation for the amplitude

$$\frac{\partial N_0(\gamma|Y)}{\partial Y} = \Gamma(1\to 2) N_0(\gamma|Y) - \Gamma(1\to 2) N_0^2(\gamma|Y) \,. \tag{14}$$

Equation (14) is easy to solve in this model and the solution has been found in [8, 89, 94]. It was noticed in [21–23] that one can get (14) directly from (11) by substitution of $N_0(\gamma|Y) = 1 - Z_0(1-\gamma|Y)$. Using this fact we can go back to (9) and write it as

$$\begin{aligned} \frac{\partial Z(1-\gamma|Y)}{\partial Y} &= \Gamma(1\to 2) \\ &\times \left(\gamma\frac{\partial Z(1-\gamma|Y)}{\partial \gamma} - \gamma^2 \frac{\partial Z(1-\gamma|Y)}{\partial \gamma}\right) . \end{aligned} \tag{15}$$

2.3 The generating functional for multiparticle production: definition and linear evolution equation

Here we would like to develop a method that will allow us not only to find the cross sections of diffractive production, but also to consider all processes of multiparticle production at high energy. Having this goal in mind, we propose a generalization of the generating functional given by (7), namely, we introduce a new generating functional $Z(u, v|Y)$ as follows:

$$Z(u, v|Y) = \sum_{n=0,m=0}^{\infty} P(n, m|Y) u^n v^m \,, \tag{16}$$

where $P(n, m|Y)$ is the probability to find n uncut pomerons and m cut pomerons. Directly from (16) and from the fact that $P(n, m|Y)$ is a probability we find the first boundary condition:

$$Z\,(u=1, v=1|Y) = 1 \,. \tag{17}$$

To find the second boundary condition we can use the full form of the s-channel unitarity constraint. Assuming that the scattering amplitude is pure imaginary at high energy, this constraint looks as follows:

$$2N(s,b) = |N(s,b)|^2 + G_{\mathrm{in}} \,, \tag{18}$$

where the first term in the l.h.s. is the elastic term with no cut pomerons, while the second one is the total contribution of the inelastic processes (in other words, a sum over all cut pomerons). Using the functionals $Z_0(u|Y)$ and $Z(u, v|Y)$ we can calculate the left and right hand sides of (18), namely [21–23]

$$N_0 = 1 - Z_0(1-\gamma|Y) \,, \tag{19}$$

$$|N(s,b)|^2 + G_{\mathrm{in}} = 2(1 - Z(1-\gamma, 1-\gamma_{\mathrm{in}}|Y)) \,, \tag{20}$$

where $\gamma = N(s=0, b)$ is the imaginary part of the scattering amplitude of a dipole with the target at low energies, while γ_{in} is the inelastic contribution ($|N(s,b)|^2 + G_{\mathrm{in}}$) for the interaction of a dipole with the target at low energy. Generally speaking both these amplitudes are arbitrary and have to be calculated from non-perturbative QCD; however, assuming that the low energies are not very low

and that we can apply (3), we see that $2\gamma = \gamma_{\rm in}$. Using this relation we can rewrite (19) and (20) in the form

$$Z_0(u|Y) = Z(u,u|Y)\,. \tag{21}$$

The initial condition depends on what we want to calculate. This is the main advantage of the generating functional, allowing us to calculate everything. For example, the cross section of single diffraction integrated over all produced masses ($\sigma_{\rm sd}$) we can find just calculating $Z(u, v = 0|Y - Y_0)$ for the initial condition in the form

$$Z\left(u, v|Y = Y_0\right) = v\,. \tag{22}$$

The cross section is equal to

$$\sigma_{\rm sd}(\gamma(Y_0)|Y - Y_0)) = 1 - Z(u = 1-\gamma, v = 0|Y - Y_0)\,. \tag{23}$$

The main idea of this paper is to introduce a cut pomeron being split into three different states. By analogy with (15) we can relate to each process a corresponding term of the differential equation

$$\not{P} \to \not{P} + \not{P} \quad \sim \quad \gamma_{\rm in}^2 \frac{\partial Z}{\partial \gamma_{\rm in}}\,, \tag{24}$$

$$\not{P} \to \not{P} + P \quad \sim \quad \gamma\gamma_{\rm in} \frac{\partial Z}{\partial \gamma_{\rm in}}\,, \tag{25}$$

$$\not{P} \to P + P \quad \sim \quad \gamma^2 \frac{\partial Z}{\partial \gamma_{\rm in}}\,; \tag{26}$$

where the notation $\not{P}$ and P stands for cut and uncut pomeron, respectively, and only second order terms in γ ($\gamma_{\rm in}$) are shown. The next step it to use the AGK cutting rules to write the resulting evolution equation

$$\begin{aligned}\frac{\partial Z}{\partial Y} &= \Gamma(1\to 2)(\gamma - \gamma^2)\frac{\partial Z}{\partial \gamma} \\ &+ \Gamma(1\to 2)\left(2\gamma^2 - 4\gamma\gamma_{\rm in} + \gamma_{\rm in}^2 + \gamma_{\rm in}\right)\frac{\partial Z}{\partial \gamma_{\rm in}}\,.\end{aligned} \tag{27}$$

It can easily be seen that the second term reproduces the first term for $2\gamma = \gamma_{\rm in}$. We note that $u = 1-\gamma$ and $v = 1-\gamma_{\rm in}$, and thus (27) for u and v reads

$$\begin{aligned}\frac{\partial Z}{\partial Y} &= -\Gamma(1\to 2)u(1-u)\frac{\partial Z}{\partial u} \\ &- \Gamma(1\to 2)(2u^2 - 4uv + v^2 + v)\frac{\partial Z}{\partial v}\,.\end{aligned} \tag{28}$$

The description in terms of a generating function becomes clear for a Markov chain. We use the definition of the generating function given by (16) to find the differential equation for the probabilities

$$\frac{\partial P\left(n,m|Y\right)}{\partial(\Gamma(1\to 2)Y)} =$$

$$(P \to P + P) \quad -nP(n,m|Y) + (n-1)P(n-1,m|Y)\,, \tag{29}$$

$$(\not{P} \to \not{P} + \not{P}) \quad +mP(n,m|Y) - (m-1)P(n,m-1|Y)\,, \tag{30}$$

$$(\not{P} \to \not{P} + P) \quad -4mP(n,m|Y) + 4mP(n-1,m|Y)\,, \tag{31}$$

$$(\not{P} \to P + P) \quad +2mP(n,m|Y) - 2(m+1) \times P(n-2,m+1|Y)\,, \tag{32}$$

where each line describes the specific process of pomeron splitting discussed above. It is instructive to compare this equation to (8). Each of (29)–(32) consists of two terms: one describes the increase of the probability to find n pomerons due to decay of one pomeron to two, and one is responsible for the decrease of this probability since one of n pomerons can decay. In all equations, except (30) and (32), the increase leads to a plus sign and a decrease to a minus sign. However, in (30) and (32) the signs are opposite in accordance with the AGK cutting rules (see Fig. 3). In this case we have to say that the decay $\not{P} \to P + P$ and $\not{P} \to \not{P} + \not{P}$ have negative amplitudes. The appearance of the minus sign in the Markov chain does not contradict the probabilistic picture since the physical meaning of the triple pomeron vertex in this framework is the rate of change of the number of pomerons. The value of the rate could be also negative, corresponding to a negative production rate (the decrease of the number of pomerons with rapidity). For a more subtle discussion of this topic the reader is referred to [78, 79], where the probabilistic interpretation of the pomeron calculus was shown to be independent of the sign of the vertices. It should be stressed that in terms of the amplitude (see (27)) we obtain positive cross sections with a different multiplicity of produced particles.

Equations (29)–(32) give a clear probabilistic interpretation of all pomeron splitting processes under discussion.

It turns out that (28), being a typical Liouville equation, has solutions that depend on the two variables $\xi_1 = \Gamma(1\to 2)Y + \ln\frac{u}{1-u}$ and $\xi_2 = \Gamma(1\to 2)Y + \ln\frac{2u-v}{1-(2u-v)}$. The general solution of this equation is given by an arbitrary function of variables ξ_1 and ξ_2, which in our case can be written as a sum of two functions, namely

$$\begin{aligned}Z &= F_1\left\{\Gamma(1\to 2)Y + \ln\frac{u}{1-u}\right\} \\ &+ F_2\left\{\Gamma(1\to 2)Y + \ln\frac{2u-v}{1-(2u-v)}\right\}.\end{aligned} \tag{33}$$

For our initial condition (22) the solution reads

$$Z = \frac{2u\mathrm{e}^{-\Gamma(1\to 2)Y}}{1 + u(\mathrm{e}^{-\Gamma(1\to 2)Y} - 1)} - \frac{(2u-v)\mathrm{e}^{-\Gamma(1\to 2)Y}}{1 + (2u-v)(\mathrm{e}^{-\Gamma(1\to 2)Y} - 1)}\,. \tag{34}$$

One can easily see that this solution satisfies both the initial condition (22) and the boundary condition (17).

2.4 The generating functional for multiparticle production: non-linear equation

Using our initial condition (22) we can rewrite the linear differential equation (28) as a non-linear one. We use the fact, mentioned above, that (28) is a differential equation of the two variables ξ_1 and ξ_2 and, thus, has no separate dependence on Y. This means that the differential equation (28) written at some rapidity Y keeps the same form for any rapidity. We pick the initial rapidity $Y = 0$ and substitute the generating function given by (22) into (28):

$$\frac{\partial Z}{\partial Y} = 0 - I\!P(1 \to 2)(2u^2 - 4uv + v^2 + v)\,. \qquad (35)$$

Now we use the initial condition from (10) for the generating function with no cut pomerons. In terms of Z_0 and Z (35) reads

$$\frac{\partial Z}{\partial Y} = -I\!P(1 \to 2)\left(2Z_0^2 - 4Z_0 Z + Z^2 + Z\right)\,. \qquad (36)$$

We identify the scattering amplitude and the diffractive cross section with $N_0(\gamma|Y) = 1 - Z_0(1-\gamma|Y)$ and $N(\gamma,\gamma_{\rm in}|Y) = 1 - Z(1-\gamma, 1-\gamma_{\rm in}|Y)$, respectively. For $N_0(\gamma|Y)$ and $N(\gamma,\gamma_{\rm in}|Y)$, (36) reads

$$\begin{aligned}\frac{\partial N(\gamma,\gamma_{\rm in}|Y)}{\partial Y} = -I\!P(1 \to 2)\big\{&2N_0^2(\gamma|Y) \\ &- 4N_0(\gamma|Y)N(\gamma,\gamma_{\rm in}|Y) \\ &+ N^2(\gamma,\gamma_{\rm in}|Y) + N(\gamma,\gamma_{\rm in}|Y)\big\}. \qquad (37)\end{aligned}$$

This equation has the same form as the equation for diffraction production[1] obtained by Kovchegov and Levin (see (5)), but it is written for a general functional and describes not only diffractive production but also the processes of particle production with any value of multiplicity. The fact that (5) has a simple generalization in the general case of QCD gives us hope to generalize this equation.

2.5 The generating functional for multiparticle production: consistency with the AGK cutting rules

We want to check the consistency of our solution (34) with the AGK cutting rules in an explicit way. To do this, we calculate the cross section of a process $\sigma^{(k)}$ with a given multiplicity k from both the generating function given by (34) and directly from the AGK cutting rules, and we compare them next. We define the cross section with multiplicity k as

$$\sigma^{(k)} = \frac{1}{k!}\frac{\partial^k N(\gamma,\gamma_{\rm in}|Y)}{\partial \gamma_{\rm in}^k}\bigg|_{\gamma_{\rm in}=0} \cdot \gamma_{\rm in}^k\,, \qquad (38)$$

where $N(\gamma,\gamma_{\rm in}|Y) = 1 - Z(1-\gamma, 1-\gamma_{\rm in}|Y)$.

[1] The difference in the overall minus sign corresponds to different definitions of the rapidity variable moving in opposite directions.

As an example we pick the multiplicity of one cut pomeron, i.e. $k = 1$ for any number of uncut pomerons. From (38) with $k = 1$ we get

$$\sigma^{(1)} = \frac{\gamma_{\rm in} e^{I\!P(1\to 2)Y}}{(1 + 2\gamma(e^{I\!P(1\to 2)Y} - 1))^2}\,. \qquad (39)$$

On the other hand we can use the coefficients for multiple pomeron exchange from the AGK cutting rules [83]. In this case the cross section for the multiplicity of k cut pomerons reads

$$\sigma^{(k)} = \sum_{n=0}^{\infty}(-1)^{n-k} C_k^n (2\gamma)^n e^{n I\!P(1\to 2)Y}\,, \qquad (40)$$

where $e^{I\!P(1\to 2)Y}$ stands for the pomeron propagator.

For $k = 1$ we sum over n in (40) and obtain

$$\sigma^{(1)} = \frac{2\gamma e^{I\!P(1\to 2)Y}}{(1 + 2\gamma e^{I\!P(1\to 2)Y})^2}\,. \qquad (41)$$

In the high energy limit $\gamma_{\rm in} = 2\gamma$ and $e^{I\!P(1\to 2)Y} - 1 \simeq e^{I\!P(1\to 2)Y}$; thus (39) reproduces (41). It can easily be shown that this holds for any value of k.

We can further compare the cross sections obtained from the generating function and a direct summation of fan diagrams using the AGK rules [96]. In our approach this corresponds to

$$\sigma_{\rm sd} = N(\gamma,\gamma_{\rm in} = 0|Y)\,, \quad \sigma_{\rm in} = N(\gamma,\gamma_{\rm in} = 2\gamma_{\rm in}|Y)\,, \qquad (42)$$

where $N(\gamma,\gamma_{\rm in}|Y) = 1 - Z(1-\gamma, 1-\gamma_{\rm in}|Y)$. The resulting expressions are identical and are given by

$$\begin{aligned}\sigma_{\rm sd} &= \frac{2\gamma^2 e^{I\!P(1\to 2)Y}(e^{I\!P(1\to 2)Y} - 1)}{(1 + \gamma(e^{I\!P(1\to 2)Y} - 1))(1 + 2\gamma(e^{I\!P(1\to 2)Y} - 1))}\,, \\ \sigma_{\rm el} &= \frac{2\gamma e^{I\!P(1\to 2)Y}}{1 + 2\gamma(e^{I\!P(1\to 2)Y} - 1)}\,, \\ \sigma_{\rm tot} &= \frac{2\gamma e^{I\!P(1\to 2)Y}}{1 + \gamma(e^{I\!P(1\to 2)Y} - 1)}\,. \qquad (43)\end{aligned}$$

3 Pomeron loops

3.1 Evolution equation with loops

Now we want to account for contributions of the pomeron loops. This problem has already been solved for the case with no cut pomerons [91]. The master equation for no cut pomerons is given by

$$\frac{\partial Z}{\partial Y} = -I\!P(1 \to 2)u(1-u)\frac{\partial Z}{\partial u} + \frac{1}{2}I\!P(2 \to 1)(u - u^2)\frac{\partial^2 Z}{\partial u^2}\,. \qquad (44)$$

Unfortunately, one cannot generalize (28) for diffractive processes by just adding second order derivative terms by

analogy with (44). The reason for this is that this type of equation would include diagrams that do not exist. For example, diagrams of the type $\not{P} \to P + P \to P$ where a cut pomeron splits to two uncut pomerons with further merging to an uncut pomeron are forbidden. To resolve this problem we introduce two separate variables w and $\bar{w}$ for an uncut pomeron in amplitude or conjugate amplitude, respectively. Naturally, these two subsets of pomerons evolve separately till the cut pomeron is introduced; alternatively, their evolutions mix only via a cut pomeron. Using our previous discussion we can readily write this new type of evolution equation based on (28):

$$\begin{aligned}\frac{\partial Z}{\partial Y} &= -\Gamma(1\to 2)\left\{w(1-w)\frac{\partial Z}{\partial w} - \bar{w}(1-\bar{w})\frac{\partial Z}{\partial \bar{w}}\right\}\\ &\quad -\Gamma(1\to 2)(2w\bar{w} - 2wv - 2\bar{w}v + v^2 + v)\frac{\partial Z}{\partial v}\\ &\quad +\frac{1}{2}\Gamma(2\to 1)\left\{(w-w^2)\frac{\partial^2 Z}{\partial w^2} + (\bar{w}-\bar{w}^2)\frac{\partial^2 Z}{\partial \bar{w}^2}\right\}\\ &\quad -\frac{1}{2}\Gamma(2\to 1)\left\{2(v-w\bar{w})\frac{\partial^2}{\partial w\partial\bar{w}} + 2(v-wv)\frac{\partial^2}{\partial w\partial v}\right.\\ &\quad \left. +\,2(v-\bar{w}v)\frac{\partial^2}{\partial\bar{w}\partial v} + (v-v^2)\frac{\partial^2}{\partial v^2}\right\} Z\,, \end{aligned} \tag{45}$$

where the first and second lines are a generalization of (28) for the variables w and $\bar{w}$, the third line corresponds to (44), and the last line needs a little bit more explanation. We have to introduce a new generating function

$$Z(w,\bar{w},v|Y) = \sum_{k=0}\sum_{l=0}\sum_{m=0} P(k,l,m|Y) w^k \bar{w}^l v^m\,, \tag{46}$$

where $P(k,l,m|Y)$ stands for the probability to find k uncut pomerons in the amplitude, l uncut pomerons in the conjugate amplitude and m cut pomerons at some rapidity Y. For the last term of (45) we write a Markov chain in a similar manner as we did for (29)–(32), namely

$$\begin{aligned}&\frac{\partial P(k,l,m|Y)}{\partial Y} =\\ &(P\to P+P) + \Gamma(1\to 2)\{(k-1)P(k-1,l,m|Y)\\ &\qquad - kP(k,l,m|Y)\}\,,\\ &(\bar{P}\to \bar{P}+\bar{P}) + \Gamma(1\to 2)\{(l-1)P(k,l-1,m|Y)\\ &\qquad - lP(k,l,m|Y)\}\,,\\ &(\not{P}\to \not{P}+ P) - \Gamma(1\to 2)\{(m-1)P(k,l,m-1|Y)\\ &\qquad - mP(k,l,m|Y)\}\,,\\ &(\not{P}\to P+ \not{P}) + 2\Gamma(1\to 2)\{mP(k-1,l,m|Y)\\ &\qquad - mP(k,l,m|Y)\}\,,\\ &(\not{P}\to \bar{P}+ \not{P}) + 2\Gamma(1\to 2)\{mP(k,l-1,m|Y)\\ &\qquad - mP(k,l,m|Y)\}\,,\\ &(\not{P}\to P+\bar{P}) - 2\Gamma(1\to 2)\{(m+1)\\ &\qquad \times P(k-1,l-1,m+1|Y)\\ &\qquad - mP(k,l,m|Y)\}\,,\\ &(P+P\to P) + \frac{1}{2}\Gamma(2\to 1)\{k(k+1)P(k+1,l,m|Y)\\ &\qquad - k(k-1)P(k,l,m|Y)\}\,,\\ &(\bar{P}+\bar{P}\to \bar{P}) + \frac{1}{2}\Gamma(2\to 1)\{l(l+1)P(k,l+1,m|Y)\\ &\qquad - l(l-1)P(k,l,m|Y)\}\,,\\ &(\not{P}+ \not{P}\to \not{P}) - \frac{1}{2}\Gamma(2\to 1)\{m(m+1)P(k,l,m+1|Y)\\ &\qquad - m(m-1)P(k,l,m|Y)\}\,,\\ &(\not{P}+P\to \not{P}) - \frac{2}{2}\Gamma(2\to 1)\{(k+1)mP(k+1,l,m|Y)\\ &\qquad - kmP(k,l,m|Y)\}\,,\\ &(\not{P}+\bar{P}\to \not{P}) - \frac{2}{2}\Gamma(2\to 1)\{(l+1)mP(k,l+1,m|Y)\\ &\qquad - lmP(k,l,m|Y)\}\,,\\ &(P+\bar{P}\to \not{P}) - \frac{2}{2}\Gamma(2\to 1)\{(k+1)(l+1)\\ &\qquad \times P(k+1,l+1,m|Y)\\ &\qquad - klP(k,l,m|Y)\}\,, \end{aligned} \tag{47}$$

where $\bar{P}$ denotes a pomeron in the conjugate amplitude and the factor of $\frac{1}{2}$ accounts for the fact that the pomerons are identical in this approach. Each line in (47) as in (29)–(32) has a clear probabilistic interpretation. We multiply (47) by $w^k\bar{w}^l v^m$, sum over all k, l and m, and using the definition (46) we obtain (45).

Even in our simple model with no coordinate dependence (45) is too complicated and its solution is still to be found. But before the solution is found we can see that putting $w = \bar{w} = u$, i.e. making no difference between uncut pomerons, the first two lines of (45) correctly reproduce (28). Moreover, we can check it further and, using the initial condition $Z(w,\bar{w},v|Y=0) = v$, we can perform iterations:

$$\frac{\partial Z_1}{\partial Y} = -\Gamma(1\to 2)(2w\bar{w} - 2wv - 2\bar{w}v + v^2 + v)\,, \tag{48}$$

giving

$$Z_1 = -\Gamma(1\to 2)(2w\bar{w} - 2wv - 2\bar{w}v + v^2 + v)Y\,. \tag{49}$$

At the next step

$$\begin{aligned} Z_2 &= +\Gamma^2(1\to 2)\\ &\quad \times\{w(1-w)(2\bar{w}-2v) - \bar{w}(1-\bar{w})(2w-2v)\}\frac{Y^2}{2}\\ &\quad +\Gamma^2(1\to 2)(2w\bar{w} - 2wv - 2\bar{w}v + v^2 + v)\\ &\quad \times(-2(w+\bar{w}) + 2v + 1)\frac{Y^2}{2}\\ &\quad -\frac{1}{2}\Gamma(1\to 2)\Gamma(2\to 1)\{2(v-w\bar{w})(-2) + 2(v-wv)2\\ &\quad +2(v-\bar{w}v)2 + (v-v^2)(-2)\}\frac{Y^2}{2}\,. \end{aligned} \tag{50}$$

It is easy to see from (50) that (45) correctly reproduces the sign and the combinatorics coefficient of the pomeron loops

in accordance with the AGK cutting rules. For example, in the first term in the third line, $-\frac{1}{2}\Gamma(1\to 2)\Gamma(2\to 1)2(v-w\bar{w})(-2)$, the term proportional to v describes the pomeron loop of the type $\not{P}\to P+\bar{P}\to\not{P}$. This loop has a factor of 2 and brings in a plus sign as expected. Similarly, the terms proportional to v in the second and third term of the last line correspond to loops of the type $\not{P}\to\not{P}+P\to\not{P}$ and $\not{P}\to\not{P}+\bar{P}\to\not{P}$, respectively. Each of them brings in a factor of -2, and putting $P=\bar{P}$ we have a factor of -4, which is in a agreement with the AGK cutting rules. Thus, we expect (45) to properly include the pomeron loops into the evolution.

At the end of this part we would like to mention that the problem has already been treated by Ciafaloni and Marchesini [82] many years ago, but in terms of the RFT Lagrangian. They separated uncut pomerons in the amplitude and conjugate amplitude by introducing different fields (ϕ_+ and ϕ_-). It is interesting to notice that, just like in our case, these authors also have partial diagonalization of the Lagrangian in terms of the new variable $\phi_+ + \phi_- - \mathrm{i}\phi_c$. By partial diagonalization one should understand the diagonal vertices only in one rapidity direction. In our case this can be formulated as a separation of the evolutions of pomerons corresponding to w, $\bar{w}$ and $w+\bar{w}-v$ in MFA.

3.2 A new method of summation of the pomeron loops: improved Mueller–Patel–Salam–Iancu approach

The solution to (45) will give the generating functional. However, (45) is based on the probabilistic interpretation of the pomeron calculus in terms of a Markov process. In this subsection we would like to suggest a different interpretation with a different procedure of summation of the pomeron diagrams.

Firstly we consider the simplest 'fan' diagram of Fig. 5. It can be calculated in an obvious way, namely

$$\begin{aligned} A(\text{Fig. 5}) &= \gamma G(Y-0) - \Delta\gamma^2\int_0^Y \mathrm{d}y_1 G(Y-y_1)G^2(y_1-0) \\ &= \gamma \mathrm{e}^{\Delta Y} - \Delta\gamma^2\int_0^Y \mathrm{d}y_1 \mathrm{e}^{\Delta(Y+y_1)} \\ &= \gamma \mathrm{e}^{\Delta Y} - \Delta\gamma^2\left(\frac{1}{\Delta}\mathrm{e}^{2\Delta Y} - \frac{1}{\Delta}\mathrm{e}^{\Delta Y}\right) \\ &= -\gamma^2\mathrm{e}^{2\Delta Y} + (\gamma+\gamma^2)\mathrm{e}^{\Delta Y} \\ &= -\gamma^2\mathrm{e}^{2\Delta Y} + \gamma_{\mathrm{R}}\mathrm{e}^{\Delta Y}\,, \end{aligned} \tag{51}$$

where $\Gamma(1\to 2)$ (see (9)) is denoted as Δ; γ is the amplitude of the pomeron interaction with the target and $G(Y-y)$ stands for the Green function of the pomeron $G(Y-y)=\exp(\Delta(Y-y))$.

As one can see, the integration over y_1 reduces the diagram in Fig. 5 to two contributions: the exchange of two non-interacting pomerons and the exchange of one pomeron with the renormalized vertex: $\gamma_{\mathrm{R}}=\gamma+\gamma^2$. In Fig. 6 is shown the pomeron 'fan' diagram of the second order, which we have to integrate over the two rapidities y_1 and y_2. The result is

$$\begin{aligned} A(\text{Fig. 6}) &= 2\Delta^2\gamma^3\int_0^Y \mathrm{d}y_1\int_0^{y_1}\mathrm{d}y_2 G(Y-y_1)G(y_1-0) \\ &\quad\times G(y_2-0)G^2(y_2-0) \\ &= 2\Delta^2\gamma^3\int_0^Y \mathrm{d}y_1\int_0^{y_1}\mathrm{d}y_2 \mathrm{e}^{\Delta(Y+y_1+y_2)} \\ &= 2\Delta^2\gamma^3\left(\frac{1}{2\Delta^2}\mathrm{e}^{3\Delta Y} - \frac{1}{\Delta^2}\mathrm{e}^{2\Delta Y} + \frac{1}{2\Delta^2}\mathrm{e}^{\Delta Y}\right). \end{aligned} \tag{52}$$

Adding the contributions of this diagram and the diagrams of Fig. 5, we obtain

$$\begin{aligned} &A(\text{Fig. 5}) + A(\text{Fig. 6}) \\ &\quad= -\gamma^3\mathrm{e}^{3\Delta Y} - (\gamma^2+2\gamma^3)\mathrm{e}^{2\Delta Y} + (\gamma+\gamma^2+\gamma^3)\mathrm{e}^{3\Delta Y} \\ &\quad= \gamma^3\mathrm{e}^{3\Delta Y} - \left(\gamma_{\mathrm{R}}^{(2)}\right)^2\mathrm{e}^{2\Delta Y} + \gamma_{\mathrm{R}}\mathrm{e}^{\Delta Y}\,. \end{aligned} \tag{53}$$

Therefore, one can see that the scattering amplitude can be rewritten as exchanges of the pomerons without interaction between them, but with a pomeron–particle vertex. In the dipole model this vertex has the meaning of the amplitude of a two dipole interaction in the Born approximation of perturbative QCD.

These two examples illustrate our main idea: pomeron calculus in zero transverse dimensions can be viewed as the theory of free, non-interacting pomerons whose interaction with the target has to be renormalized. It is easy to see that in the MFA we can rewrite the master equation (see (9) and (13)) in the form

$$\frac{\partial N_0(\gamma_{\mathrm{R}}|Y)}{\partial Y} = \Gamma(1\to 2)\gamma_{\mathrm{R}}\frac{\partial N_0(\gamma_{\mathrm{R}}|Y)}{\partial \gamma_{\mathrm{R}}}\,, \tag{54}$$

with

$$\gamma_{\mathrm{R}} = \frac{\gamma}{1-\gamma}\,. \tag{55}$$

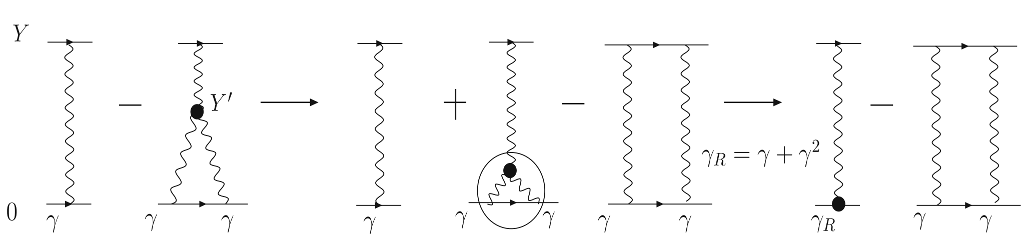

Fig. 5. The renormalization procedure in the case of the simplest 'fan' diagram

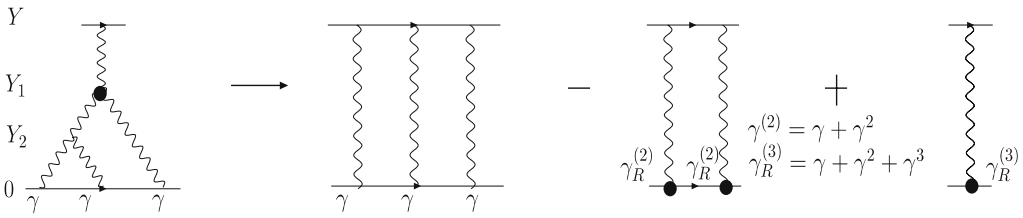

Fig. 6. The renormalization procedure in the case of the 'fan' diagram of the second order

We have shown the way that (55) has started to build up in a perturbation expansion in (51) and (52).

The general solution of (54) is a system of non-interacting pomerons, and the scattering amplitude can be found in the form

$$N_0(\gamma_{\rm R}|Y) = \sum_{n=1}^{\infty} C_n \gamma_{\rm R}^n G^n(Y-0)\,, \tag{56}$$

where the coefficients C_n could be found from the initial conditions, namely from the expression for the low energy amplitude. In particular, the initial condition

$$N_0(\gamma_{\rm R}|Y=0) = \gamma = \gamma_{\rm R}/(1+\gamma_{\rm R}) \tag{57}$$

generates $C_n = (-1)^n$ and the solution is

$$N_0(\gamma_{\rm R}|Y) = \frac{\gamma_{\rm R} e^{\Delta Y}}{1+\gamma_{\rm R} e^{\Delta Y}}\,. \tag{58}$$

The initial condition of (57) has very simple physics behind it. It describes the independent (non-correlated) production of the pomerons at low energy but with only the condition that one pomeron lives a shorter time than the second one (see Fig. 7). If all n pomerons were emitted by the same dipole (see Fig. 7b) this condition leads to a Glauber factor $1/n!$ leading to $N_0(\gamma_{\rm R}|Y=0) = \gamma = 1-\exp(-\gamma_{\rm R})$ instead of (57). However, if pomerons are produced as the consequent decays (see Fig. 7a) the factor is equal to 1. The factor $(-1)^n$ comes from Glauber screening, resulting in (57). In QCD we have strong evidence that the second case is correct [60–64].

For the analysis of enhanced diagrams we start from the first diagram of Fig. 8. It leads to the following contribution:

$$\begin{aligned} A(\text{Fig. 8}) &= -\Delta^2\gamma^2 \int_0^Y {\rm d}y_1 \int_0^{y_1} {\rm d}y_2 G(Y-y_1) \\ &\quad \times G^2(y_1-y_2) G(y_2-0) \\ &= -\Delta^2\gamma^2 \int_0^Y {\rm d}y_1 \int_0^{y_1} {\rm d}y_2\, {\rm d}^{\Delta(Y+y_1-y_2)} \\ &= -\gamma^2 {\rm e}^{2\Delta Y} + \gamma^2 {\rm e}^{\Delta Y} + \Delta\gamma^2 Y {\rm e}^{\Delta Y}\,, \end{aligned} \tag{59}$$

where $\Gamma(2\to 1) = \Delta\gamma^2$ [54, 55].

Adding (59) to the exchange of the one pomeron, we obtain the result that the exchange of one pomeron and the enhanced diagram of Fig. 8 can be written in close form

$$\text{One pomeron exchange} + A(\text{Fig. 8}) = \gamma_{\rm R} {\rm e}^{\Delta_{\rm R} Y} - \gamma^2 {\rm e}^{2\Delta Y}\,, \tag{60}$$

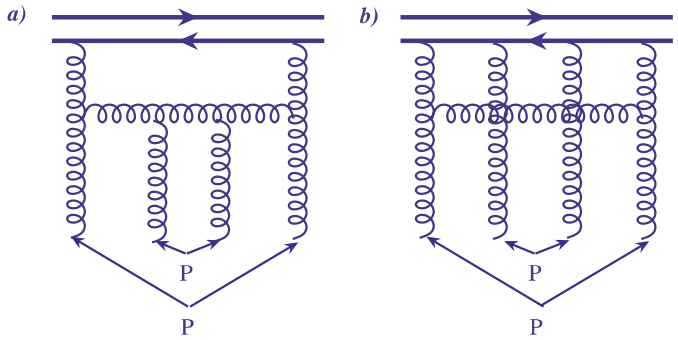

Fig. 7. The diagram that illustrates the initial condition of (57)

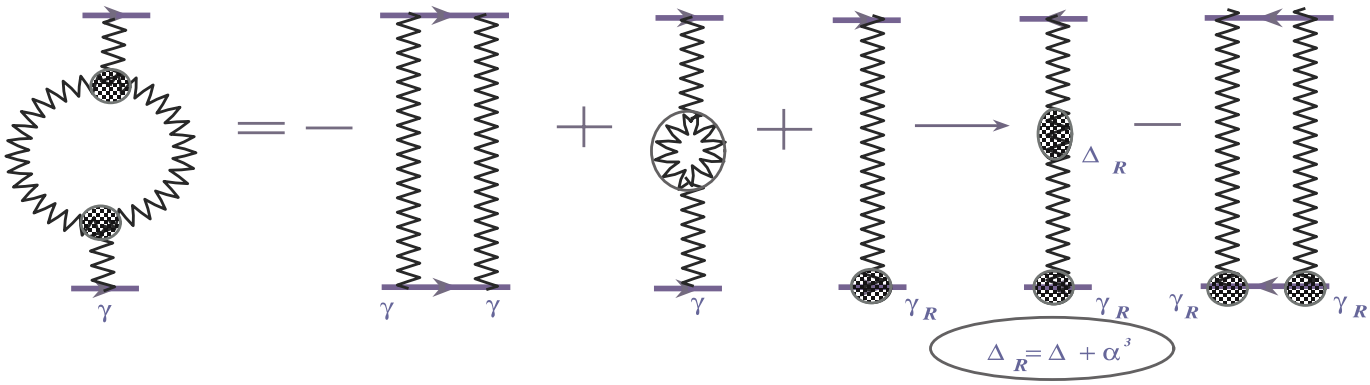

Fig. 8. The renormalization procedure in the case of the simplest enhanced diagram

with

$$\gamma_{\rm R} = \gamma^{(2)} = \gamma + \gamma^2\,, \quad \Delta_{\rm R} = \Delta + \gamma\Delta\,. \tag{61}$$

It is easy to see that (59) can be viewed as the expansion to first order of (60).

Therefore, the pomeron loops can be either large (of the order of Y) and they can be considered as un-enhanced diagrams; or they may be small (of the order of $1/\Delta$) and can be treated as the renormalization of the pomeron intercept.

In QCD $\Delta \propto \bar{\alpha}_{\rm S}$, while $\gamma \propto \alpha_{\rm S}^2$. Therefore, the renormalization of the pomeron intercept Δ is proportional to $\alpha_{\rm S}^3$. We can neglect this contribution since (i) there are a lot of $\alpha_{\rm S}^2$ corrections to the kernel of the equation that are much larger than this contribution; and (ii) in the region of $Y \ll 1/\alpha_{\rm S}^2$, where we can trust our pomeron calculus (see the introduction), we have $(\Delta_{\rm R} - \Delta)Y \ll 1$.

As a next step in our proof we have to consider the reduction to a system of non-interacting pomerons of the more complicated enhanced diagrams shown in Fig. 9. One

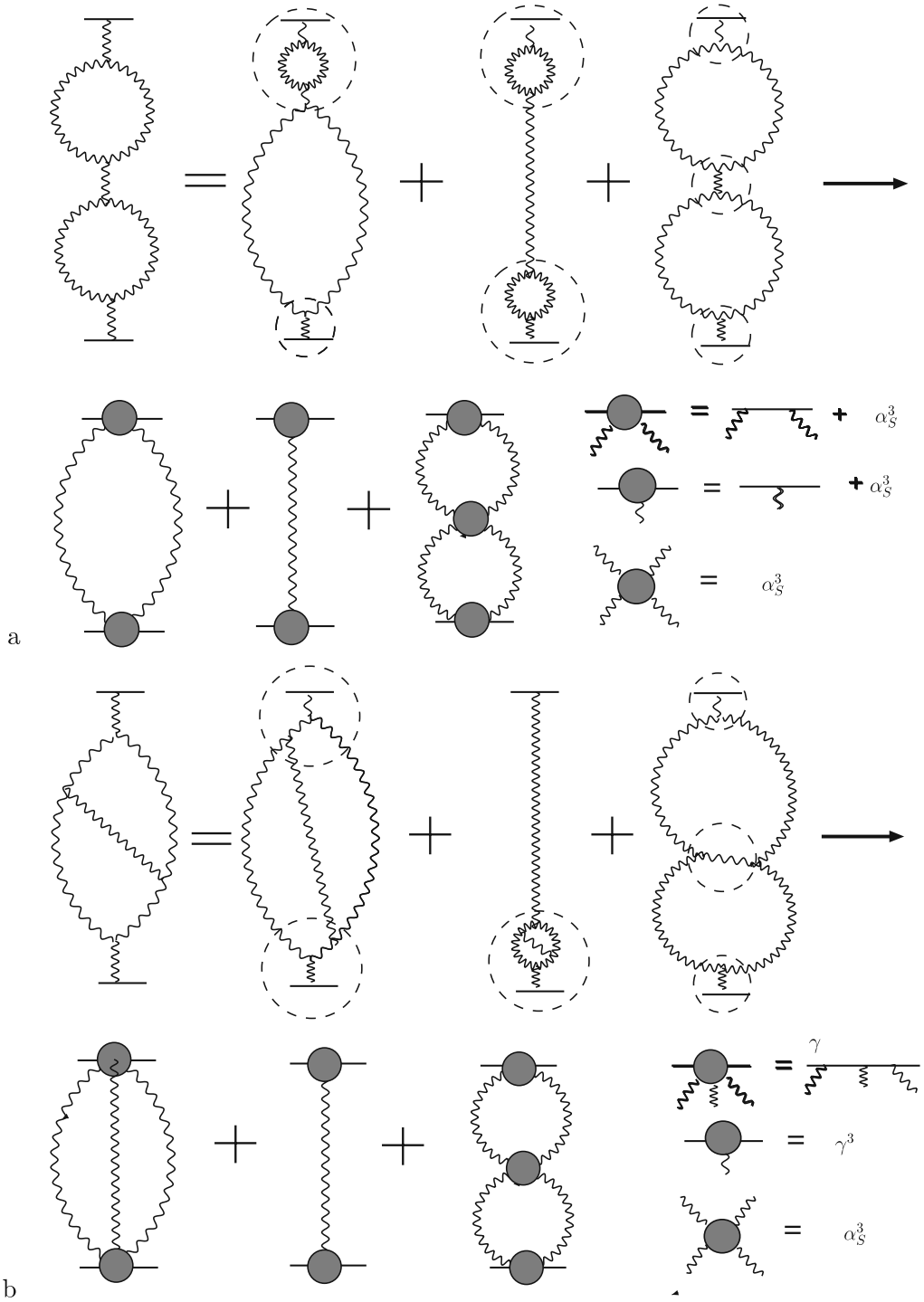

Fig. 9. More examples of the reduction of enhanced diagrams to the system of non-interacting pomerons

can see from this figure that the small pomeron loops lead to a contribution of the order of α_S^3 to the renormalization of the vertices of the particle–pomerons interaction and can be ignored. The pomeron interaction also gives the new intercept for the system of n pomerons ($n = 2$ in Fig. 9), but this intercept turns out to be of the order of α_S^3 and can be safely ignored in the kinematical region we consider here.

For a general proof that the enhanced diagrams can be reduced to a system of non-interacting pomerons it is convenient to write the diagrams in the ω representation. In this representation the single pomeron exchange has the following form:

$$G_P(Y) = e^{\Delta Y} = \int_{a-i\infty}^{a+i\infty} \frac{d\omega}{2\pi i} \frac{e^{\omega Y}}{\omega - \Delta}. \tag{62}$$

A general enhanced diagram (see Fig. 10) can be written in the form

$$A_n = \gamma \int_{a-i\infty}^{a+i\infty} \frac{d\omega}{2\pi i} \frac{1}{(\omega-\Delta)^n} e^{\omega Y} \Sigma^{n-1}(1PI), \tag{63}$$

where $\Sigma(1PI)$ is an arbitrary pomeron diagram that does not contain one pomeron exchange. This one pomeron irreducible diagram can be presented in the form

$$\Sigma(1PI) = \frac{\Delta^2\gamma}{\omega - 2\Delta} + \left(\frac{\Delta^2\gamma}{\omega-2\Delta}\right)^2 \prod_{i=1, j_i \geq 1} \frac{\Delta^2\gamma}{\omega - j_i\Delta}, \tag{64}$$

where $\Delta^2\gamma = \Gamma(1 \to 2)\Gamma(2 \to 1) \simeq \alpha_S^3$ in our normalization. The integral over ω in (63) can be closed on the pole Δ of the mth order or some pole in $\Sigma(1PI)$. In the former case one obtains the contribution (see the first diagram in Fig. 10)

$$A_n(\omega = \Delta) = \frac{1}{(n-1)!} \frac{\partial^{n-1}}{\partial \omega^{n-1}} (\Sigma(1PI)^{n-1} e^{\omega Y}) \Big|_{\omega=\Delta}. \tag{65}$$

Since $\Sigma(1PI)$ contains at least one factor of $\frac{1}{\Delta - 2\Delta}$ that is of the order of α_S^3 at $\omega = \Delta$, the only contribution that is not small at $\alpha_S Y \leq 1/\alpha_S$ is the exchange of a single pomeron (a term with $n = 1$ in (65)). Closing on singularities of $\Sigma(1PI)$ we can extract the two pomeron irreducible diagram (the second diagram in Fig. 10). Repeating the same procedure as we did for single pomeron exchange (see the fourth and fifth diagram in Fig. 10) we conclude that the corrections to the intercept of the two pomerons are also small and of the order of α_S^3. The reason for this is the same since $\Sigma(2PI)$ includes at least one factor of $\frac{1}{\Delta - 2\Delta}$ (see Fig. 9a) or a factor of $\frac{1}{2\Delta - 3\Delta}$ as in Fig. 9b. Both factors are of the same order and can be neglected. Applying the same procedure to the diagrams in the last term in Fig. 10 we introduce the three pomeron irreducible diagram and extract the term with exchange of three pomerons. From this discussion one can see that in the region $\alpha_S Y \leq 1/\alpha_S$ the leading contribution comes from big loops corresponding to the exchange of free pomerons. This implies that we can safely use the t-channel unitarity condition to match the two tree diagram contributions in the opposite direction in rapidity (Mueller–Patel–Salam–Iancu prescription) to properly account for all loops in our region $\alpha_S Y \leq 1/\alpha_S$ as will be shown in the next chapter. Strong evidence of the applicability of the Mueller–Patel–Salam–Iancu prescription was demonstrated in [84], where diagrams were compared numerically and the MPSI prescription showed very good agreement with the exact solution.

Concluding this analysis we can claim that pomeron calculus in zero dimensions is a theory of non-interacting pomerons with renormalized vertices of the pomeron–particle interaction. In dipole language, it means that we have a system of non-interacting pomerons with a specific hypothesis on the amplitude of the dipole interactions at low energy. For the problem that we are solving here, namely when we have one bare pomeron at low ene rgy, this amplitude is determined by (57).

For such a system we can calculate the scattering amplitude using a method suggested by Mueller, Patel, Salam and Iancu and developed in a number of papers (see [7, 10, 14, 79, 85–88, 95, 97] and references therein). This method suggests that the scattering amplitude can be calculated using the t-channel unitarity constraint, which is written in the following way (assuming that amplitudes

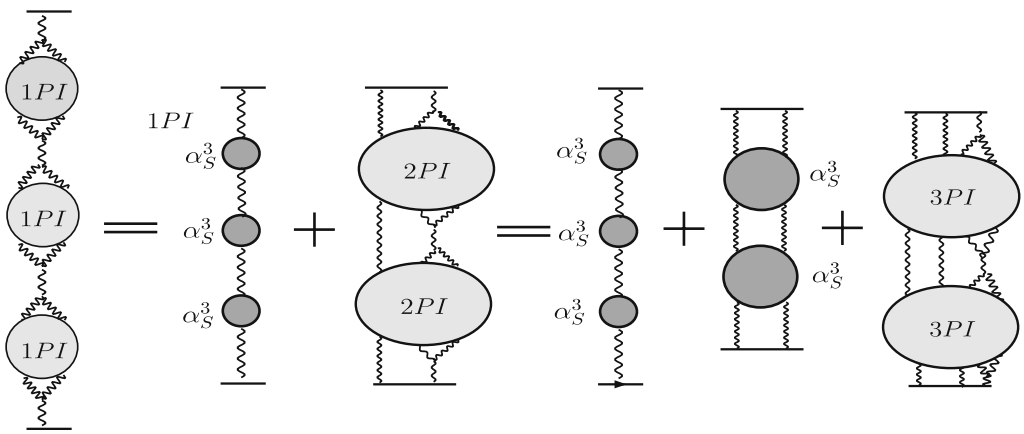

Fig. 10. The graphic representation of the general proof that the enhanced diagrams can be reduced to a system of non-interacting pomerons

at high energy are pure imaginary, $N = \mathrm{Im}\, A$):

$$N([\dots]|Y) = N([\dots]|Y - Y'; P \to nP) \bigotimes N([\dots]|Y'; P \to nP)\,, \qquad (66)$$

where $\bigotimes$ stands for all needed integrations while $[\dots]$ describes all quantum numbers (dipole sizes and so on).

The correct implementation of all this leads in our case to the following formula [14, 95, 97]:

$$N_0^{\mathrm{MPSI}}(Y) = 1 - \exp\left\{-\gamma^{\mathrm{BA}} \frac{\partial}{\partial \gamma_{\mathrm{R}}^1} \frac{\partial}{\partial \gamma_{\mathrm{R}}^2}\right\} \times N^{\mathrm{MFA}}\left(\gamma_{\mathrm{R}}^1 | Y - Y'\right) \times N^{\mathrm{MFA}}\left(\gamma_{\mathrm{R}}^2 | Y'\right)\big|_{\gamma_{\mathrm{R}}^1 = \gamma_{\mathrm{R}}^2 = 0}\,, \qquad (67)$$

where $N^{\mathrm{MFA}}(Y, \gamma_{\mathrm{R}})$ is given by (58) (see also (54)) in the mean field approximation and $\gamma^{\mathrm{BA}} \propto \alpha_{\mathrm{S}}^2$ is the scattering amplitude at low energies, which is described by the Born approximation in perturbative QCD. The difference of (67) from the original MPSI approach is the fact that this equation does not depend on the value of Y' and, because of this, we do not need to choose $Y' = Y/2$ for the best accuracy.

Substituting (58) in (67) we obtain

$$N_0^{\mathrm{MPSI}}\left(\gamma^{\mathrm{BA}} | Y\right) = 1 - \exp\left(\frac{1}{\gamma^{\mathrm{BA}} \mathrm{e}^{\Gamma(1\to 2)Y}}\right) \times \frac{1}{\gamma^{\mathrm{BA}} \mathrm{e}^{\Gamma(1\to 2)Y}} \Gamma\left(0, \frac{1}{\gamma^{\mathrm{BA}} \mathrm{e}^{\Gamma(1\to 2)Y}}\right). \qquad (68)$$

$\Gamma(0, x)$ is the incomplete gamma function (see formulae 8.350–8.359 in [98]).

We claim that (68) is the solution to our problem. One can easily see that $N_0(\gamma|Y) \to 1$ at high energies, in contrast to the exact solution with the Hamiltonian of (1). The exact solution leads to an amplitude that vanishes at high energy [1–6, 13]. As has been mentioned the solution depends crucially on the initial condition for the scattering amplitude at low energies. For (68) this amplitude is equal to

$$N_0^{\mathrm{MPSI}}\left(\gamma | Y = 0\right) = \sum_{n=1}^{\infty} (-1)^{n+1} n! (\gamma^{\mathrm{BA}})^n\,, \qquad (69)$$

with $\gamma^{\mathrm{BA}} \propto \alpha_{\mathrm{S}}^2$. This equation reminds us of the ultraviolet renormalon contribution and calls for better understanding of the non-perturbative contribution.

We can rewrite (67) in a more convenient form using the Cauchy formula for the derivatives, namely

$$\frac{\partial^n Z^{\mathrm{MFA}}(\gamma_{\mathrm{R}}|Y)}{\partial \gamma_{\mathrm{R}}^n} = n! \frac{1}{2\pi \mathrm{i}} \oint_C \frac{Z^{\mathrm{MFA}}(\gamma'_{\mathrm{R}}|Y)}{\gamma_{\mathrm{R}}'^{n+1}} \mathrm{d}\gamma'_{\mathrm{R}}\,. \qquad (70)$$

The contour C in (70) is a circle with a small radius around $\gamma_{\mathrm{R}} = 0$. However, since the function Z does not grow at large γ_{R} for $n \le 1$, we can close our contour C on the singularities of the function Z. We will call this new contour C_{R}.

We have

$$\begin{aligned} N_0^{\mathrm{MPSI}}(Y) &= 1 - \exp\left\{-\gamma^{\mathrm{BA}} \frac{\partial}{\partial \gamma_{\mathrm{R}}^1} \frac{\partial}{\partial \gamma_{\mathrm{R}}^2}\right\} N^{\mathrm{MFA}}(\gamma_{\mathrm{R}}^1 | Y - Y') \\ &\quad \times N^{\mathrm{MFA}}(\gamma_{\mathrm{R}}^2 | Y')\big|_{\gamma_{\mathrm{R}}^1 = \gamma_{\mathrm{R}}^2 = 0} \\ &= 1 - \sum_{n=1}^{\infty} \frac{(-\gamma^{\mathrm{BA}})^n}{n!} n! n! \frac{1}{(2\pi \mathrm{i})^2} \oint_{C_{\mathrm{R}}^1} \mathrm{d}\gamma_{\mathrm{R}}^1 \frac{Z^{\mathrm{MFA}}(\gamma_{\mathrm{R}}^1 | Y - Y')}{(\gamma_{\mathrm{R}}^1)^{n+1}} \\ &\quad \times \oint_{C_{\mathrm{R}}^2} \mathrm{d}\gamma_{\mathrm{R}}^2 \frac{Z^{\mathrm{MFA}}(\gamma_{\mathrm{R}}^2 | Y')}{(\gamma_{\mathrm{R}}^2)^{n+1}} \\ &= \frac{1}{(2\pi \mathrm{i})^2} \oint \oint \frac{\mathrm{d}\tilde{\gamma}_{\mathrm{R}}^1}{\tilde{\gamma}_{\mathrm{R}}^1} \frac{\mathrm{d}\tilde{\gamma}_{\mathrm{R}}^2}{\tilde{\gamma}_{\mathrm{R}}^2} \left\{1 - \exp\left(\frac{\tilde{\gamma}_{\mathrm{R}}^1 \tilde{\gamma}_{\mathrm{R}}^2}{\gamma^{\mathrm{BA}} \mathrm{e}^{\Gamma(1\to 2)Y}}\right) \right. \\ &\quad \left. \times \frac{\tilde{\gamma}_{\mathrm{R}}^1 \tilde{\gamma}_{\mathrm{R}}^1}{\gamma^{\mathrm{BA}} \mathrm{e}^{\Gamma(1\to 2)Y}} \Gamma\left(0, \frac{\tilde{\gamma}_{\mathrm{R}}^1 \tilde{\gamma}_{\mathrm{R}}^2}{\gamma^{\mathrm{BA}} \mathrm{e}^{\Gamma(1\to 2)Y}}\right)\right\} \\ &\quad \times Z^{\mathrm{MFA}}\left(\tilde{\gamma}_{\mathrm{R}}^1\right) Z^{\mathrm{MFA}}\left(\tilde{\gamma}_{\mathrm{R}}^2\right). \end{aligned} \qquad (71)$$

Here we introduce the new variables $\tilde{\gamma}_{\mathrm{R}}^1 = \gamma_{\mathrm{R}}^1 \exp(\Gamma(1\to 2)(Y - Y'))$ and $\tilde{\gamma}_{\mathrm{R}}^2 = \gamma_{\mathrm{R}}^2 \exp(\Gamma(1\to 2)Y')$. In these new variables

$$Z^{\mathrm{MFA}}\left(\tilde{\gamma}_{\mathrm{R}}^1\right) = \frac{1}{1 + \tilde{\gamma}_{\mathrm{R}}^1}\,, \quad Z^{\mathrm{MFA}}\left(\tilde{\gamma}_{\mathrm{R}}^2\right) = \frac{1}{1 + \tilde{\gamma}_{\mathrm{R}}^2}\,. \qquad (72)$$

Closing the integration on the poles $\tilde{\gamma}_{\mathrm{R}}^1 = -1$ and $\tilde{\gamma}_{\mathrm{R}}^2 = -1$, we obtain (68).

3.3 The generating functional for multiparticle production with pomeron loops

Using the result of the previous section we will derive the formula in the MPSI approach for the general functional, defined by (46). This formula is based on the solution of (45) but without the secondary derivatives. Such a solution gives the MFA approximation to our problem and we denote it as $Z^{\mathrm{MFA}}(w, \bar{w}, v|Y)$. The equation for $Z^{\mathrm{MFA}}(w, \bar{w}, v|Y)$ looks as follows:

$$\begin{aligned} Z(w, \bar{w}, v|Y) &= \frac{w \mathrm{e}^{-\Gamma(1\to 2)Y}}{1 + w(\mathrm{e}^{-\Gamma(1\to 2)Y} - 1)} \\ &\quad + \frac{\bar{w} \mathrm{e}^{-\Gamma(1\to 2)Y}}{1 + \bar{w}(\mathrm{e}^{-\Gamma(1\to 2)Y} - 1)} \\ &\quad - \frac{(w + \bar{w} - v)\mathrm{e}^{-\Gamma(1\to 2)Y}}{1 + (w + \bar{w} - v)(\mathrm{e}^{-\Gamma(1\to 2)Y} - 1)}\,. \end{aligned} \qquad (73)$$

Using the renormalized γ of (55) we can rewrite (73) in a different form, namely

$$\begin{aligned} &Z^{\mathrm{MFA}}(\gamma_{\mathrm{R}}, \bar{\gamma}_{\mathrm{R}}, \gamma_{\mathrm{in,R}}|Y) \\ &\quad = \frac{1}{1 + \gamma_{\mathrm{R}} \mathrm{e}^{\Gamma(1\to 2)Y}} + \frac{1}{1 + \bar{\gamma}_{\mathrm{R}} \mathrm{e}^{\Gamma(1\to 2)Y}} - \frac{1}{1 + \xi_{\mathrm{R}} \mathrm{e}^{\Gamma(1\to 2)Y}} \\ &\quad = \frac{1}{1 + \tilde{\gamma}_{\mathrm{R}}} + \frac{1}{1 + \tilde{\bar{\gamma}}_{\mathrm{R}}} - \frac{1}{1 + \tilde{\xi}_{\mathrm{R}}}\,, \end{aligned} \qquad (74)$$

where we write $\xi = 1 - w - \bar{w} + v = \gamma + \bar{\gamma} - \gamma_{\rm in}$ and

$$\begin{aligned}
&\xi_{\rm R} \equiv \gamma_{\rm R} + \bar{\gamma}_{\rm R} - \gamma_{\rm in,R} = \frac{\xi}{1-\xi}\,, \quad \xi = \frac{\xi_{\rm R}}{1+\xi_{\rm R}}\,,\\
&\tilde{\gamma}_{\rm R} = \gamma_{\rm R} e^{I\!\Gamma(1\to 2)Y}\,; \quad \tilde{\bar{\gamma}}_{\rm R} = \bar{\gamma}_{\rm R} e^{I\!\Gamma(1\to 2)Y}\,,\\
&\tilde{\xi_{\rm R}} = \xi_{\rm R} e^{I\!\Gamma(1\to 2)Y}\,.
\end{aligned} \tag{75}$$

The first equation of (75) is the definition of $\gamma_{\rm in,R}$.

The general formula for the amplitude in the MPSI approach has the form

$$\begin{aligned}
&N^{\rm MPSI}\left(\gamma^{\rm BA}, \gamma^{\rm BA}_{\rm in}|Y\right)\\
&= \left(\exp\left\{-\gamma^{\rm BA}\frac{\partial}{\partial\gamma^1_{\rm R}}\frac{\partial}{\partial\gamma^2_{\rm R}} - \gamma^{\rm BA}\frac{\partial}{\partial\bar{\gamma}^1_{\rm R}}\frac{\partial}{\partial\bar{\gamma}^2_{\rm R}}\right.\right.\\
&\quad \left.\left. + \gamma^{\rm BA}_{\rm in}\frac{\partial}{\partial\gamma^1_{\rm in,R}}\frac{\partial}{\partial\gamma^2_{\rm in,R}}\right\} - 1\right)\\
&\quad \times Z^{\rm MFA}\left(\gamma^1_{\rm R}, \bar{\gamma}^1_{\rm R}, \gamma^1_{\rm in,R}\big|\, Y - Y'\right)\\
&\quad \times Z^{\rm MFA}\left(\gamma^2_{\rm R}, \bar{\gamma}^2_{\rm R}, \gamma^2_{\rm in,R}|Y'\right)\Big|_{\gamma^1_{\rm R}=\gamma^2_{\rm R}=\bar{\gamma}^1_{\rm R}=\bar{\gamma}^2_{\rm R}=\gamma^1_{\rm in,R}=\gamma^2_{\rm in,R}=0}\\
&= \left(\exp\left\{-\gamma^{\rm BA}\frac{\partial}{\partial\tilde{\gamma}^1_{\rm R}}\frac{\partial}{\partial\tilde{\gamma}^2_{\rm R}} - \gamma^{\rm BA}\frac{\partial}{\partial\tilde{\bar{\gamma}}^1_{\rm R}}\frac{\partial}{\partial\tilde{\bar{\gamma}}^2_{\rm R}}\right.\right.\\
&\quad \left.\left. + \gamma^{\rm BA}_{\rm in}\frac{\partial}{\partial\tilde{\gamma}^1_{\rm in,R}}\frac{\partial}{\partial\tilde{\gamma}^2_{\rm in,R}}\right\} - 1\right)\\
&\quad \times Z^{\rm MFA}\left(\tilde{\gamma}^1_{\rm R}, \tilde{\bar{\gamma}}^1_{\rm R}, \tilde{\gamma}^1_{\rm in,R}\right)\\
&\quad \times Z^{\rm MFA}\left(\tilde{\gamma}^2_{\rm R}, \tilde{\bar{\gamma}}^2_{\rm R}, \tilde{\gamma}^2_{\rm in,R}\right)\Big|_{\gamma^1_{\rm R}=\gamma^2_{\rm R}=\bar{\gamma}^1_{\rm R}=\bar{\gamma}^2_{\rm R}=\gamma^1_{\rm in,R}=\gamma^2_{\rm in,R}=0}\,,
\end{aligned} \tag{76}$$

where $\gamma^{\rm BA}$ and $\gamma^{\rm BA}_{\rm in}$ are the elastic and inelastic amplitudes of the interaction of two dipoles at low energy, which are calculated in QCD in the Born approximation. The plus sign in (76) in front of $\gamma^{\rm BA}_{\rm in}\frac{\partial}{\partial\tilde{\gamma}^1_{\rm in,R}}\frac{\partial}{\partial\tilde{\gamma}^2_{\rm in,R}}$ reflects the fact that the pomeron loop with two cut pomerons does not have a negative contribution unlike the case of uncut pomerons. The sign between the exponent and unity in (76) could easily be checked noticing that the first term of the expansion of the exponent correctly reproduces the positive contribution of $\gamma_{\rm in}$ (cut pomeron).

The nice feature of this equation that one can see is that the result does not depend on the value of the arbitrarily chosen rapidity Y'. Using the explicit form for $Z^{\rm MFA}$ given by (74) we can calculate $Z^{\rm MPSI}$ in closed form denoting

$$G(x) \equiv \exp\left(\frac{1}{x}\right)\frac{1}{x}\Gamma\left(0, \frac{1}{x}\right). \tag{77}$$

$N^{\rm MPSI}$ is equal to

$$\begin{aligned}
&N^{\rm MPSI}\left(\gamma^{\rm BA}, \gamma^{\rm BA}_{\rm in}|Y\right)\\
&\quad = 2\left(1 - G\left(\gamma^{\rm BA} e^{I\!\Gamma(1\to 2)Y}\right)\right)\\
&\quad\quad - \left(1 - G\left(\left(2\gamma^{\rm BA} - \gamma^{\rm BA}_{\rm in}\right) e^{I\!\Gamma(1\to 2)Y}\right)\right).
\end{aligned} \tag{78}$$

Useful formulae for getting (78) are the following:

$$\frac{\partial^k}{\partial\gamma^k_{\rm in}}\frac{\partial^{l_1}}{\partial\gamma^{l_1}}\frac{\partial^{l_2}}{\partial\bar{\gamma}^{l_2}}\frac{1}{1+\gamma+\bar{\gamma}-\gamma_{\rm in}} = (-1)^{l_1+l_2}(l_1+l_2+k)! \tag{79}$$

and equations 8.350–8.359 for the incomplete gamma function $\Gamma(0, x)$ in [98].

Equation (78) allows us to calculate the cross section with fixed multiplicity of produced particles. Namely, the cross section for the processes with $k\langle n\rangle$ particles in the final state, where $\langle n\rangle$ is the mean multiplicity in our reaction, can be calculated as

$$\begin{aligned}
\sigma_k(Y) = \frac{1}{k!}&\left(\frac{\partial^k}{\partial\left(\gamma^{\rm BA}_{\rm in}\right)^k} N^{\rm MPSI}\left(\gamma^{\rm BA}, \gamma^{\rm BA}_{\rm in}|Y\right)\right)\Big|_{\gamma^{\rm BA}_{\rm in}=0}\\
&\cdot\left(\gamma^{\rm BA}_{\rm in} = 2\gamma^{\rm BA}\right)^k.
\end{aligned} \tag{80}$$

Here we use that $\gamma^{\rm BA}_{\rm in} = 2\gamma^{\rm BA}$ in the Born approximation of QCD.

It is interesting to check the general equation (see (78)) calculating two known cases: the diffractive dissociation process and the total inelastic cross section. The first one can be calculated using (78) with $\gamma^{\rm BA}_{\rm in} = 0$. The answer is

$$N^{\rm MPSI}_{\rm diff}(\gamma^{\rm BA}|Y) = 2N^{\rm MPSI}_0(\gamma^{\rm BA}|Y) - N^{\rm MPSI}_0(2\gamma^{\rm BA}|Y)\,, \tag{81}$$

where N_0 is given by (68). Equation (81) is a direct consequence of the unitarity constraints (see (18)). As one can see from (34) the same formula determines diffractive production in the mean field approximation.

The value of the total inelastic cross section, which is equal to the sum of diffractive production and inelastic cross section, stems from (78) for $\gamma^{\rm BA}_{\rm in} = 2\gamma^{\rm BA}$, and it is equal to

$$\begin{aligned}
N^{\rm MPSI}_{\text{total inelastic}}(\gamma^{\rm BA}|Y) &= 2N^{\rm MPSI}_0(\gamma^{\rm BA}|Y)\\
&= N^{\rm MPSI}_{\rm diff}(\gamma^{\rm BA}|Y) + N^{\rm MPSI}_{\rm inel}(\gamma^{\rm BA}|Y)\,,
\end{aligned} \tag{82}$$

which is actually the unitarity constraint itself (see (18)). One can see that the inelastic cross section is determined by

$$N^{\rm MPSI}_{\rm inel}(\gamma^{\rm BA}|Y) = 1 - G(2\gamma^{\rm BA} e^{I\!\Gamma(1\to 2)Y})\,. \tag{83}$$

4 Conclusions

In this paper we introduce a new generating function for the processes of multiparticle production both for the mean field approximation (see (16)) and for the general case (see (46)). For the general case where the pomeron loops have been taken into account, we obtain the linear evolution equation for the generating function (see (45)), while in the mean field approximation we proved both

the linear evolution equation (see (28)) and the non-linear equation (see (36)). The latter is the generalization of the Kovchegov–Levin equation for diffractive production to the general case of processes with arbitrary multiplicities. Since this equation is proven for the general QCD case we hope that the equations for the general generating function can be proven for the real QCD evolution.

The second result of the paper is a new method of summing the pomeron loops. We argued that the sum of all pomeron diagrams, including loops, in the kinematic region of (2) can be reduced to the diagrams of pomeron exchanges without interactions between the pomerons if we renormalize the amplitude of the low energy interaction. Based on this result we suggest a generalization of the Mueller–Patel–Salam–Iancu method of summation of the pomeron loops. In particular, we calculated the new generating function for the inelastic processes in the improved MPSI approximation (see (78)).

We would like to stress that we firmly believe that the scattering amplitude calculated using this method leads to a correct answer to the old problem: the high energy asymptotic behavior of the scattering amplitude at ultra high energies beyond pomeron calculus in the kinematic region of (2).

We hope that both results will lead to new simplifications in the case of BFKL pomeron calculus in QCD (in two transverse dimensions). The general case of BFKL pomeron calculus in QCD will be addressed in a separate paper. We would like also to mention that a discussion of this case has started in [99–102].

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